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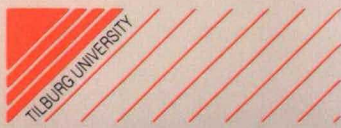


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by
Helmut Bester

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BERTRAND EQUILIBRIUM IN A DIFFERENTIATED DUOPOLY*

BY HELMUT BESTER¹

This paper studies the stability of price competition in a horizontally differentiated duopoly. The firms' demand is derived from a distribution of consumer preferences. This description of the consumer sector is applicable to a large class of differentiated commodity markets, including spatial competition models. We show that there is a (pure) price setting equilibrium when consumer tastes are sufficiently dispersed. Further conditions on the dispersedness of preferences guarantee uniqueness of the equilibrium. In addition, we examine the relation between consumer preferences and the competitiveness and efficiency of the equilibrium outcome.

1. INTRODUCTION

This paper investigates the stability of price competition in a horizontally differentiated duopoly. The duopolists' demand is derived from a distribution of preference characteristics over the population of consumers. We show that competition between the firms results in a (pure) price equilibrium when consumer tastes are sufficiently dispersed. The competitiveness of the equilibrium is closely related to the diversity of consumer types. When the support of the preference distribution shrinks to a single point, the equilibrium approaches the Bertrand outcome of a homogeneous good market. We further show that a sufficient degree of preference dispersion guarantees uniqueness of the equilibrium. Finally, we discuss the firms' incentives for product differentiation from the viewpoint of social efficiency.

The attractiveness of Bertrand's (1883) approach to the theory of oligopoly lies in the fact that in his model prices are chosen by economic agents rather than by a fictitious auctioneer. Yet, modelling price competition leads to a number of problems, especially with homogeneous goods. As already Bertrand (1883) observed, in the case of equally efficient firms and constant marginal costs the price setting equilibrium coincides with the competitive outcome. This extreme prediction, that holds even with only two firms, appears paradoxical and economically uninteresting for oligopolistic competition. In the absence of the constant returns to scale assumption, the Bertrand model faces up to a further drawback, namely the problem of nonexistence of equilibrium. This problem was first pointed out by Edgeworth (1925) in his analysis of a capacity constrained oligopoly. In his famous article on the stability of competition, Hotelling (1929) took the view that these problems originate in the abstraction of homogeneous goods. Under this assump-

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tion each single firm can attract the whole market by only slightly undercutting the prices of its rivals. Hotelling argued that this "leads to a type of instability which disappears when the quantity sold by each (seller) is considered as a continuous function of the differences in price" (Hotelling 1929, p. 44). Unfortunately, however, restoring continuity of demand by introducing product heterogeneity is not sufficient to guarantee the existence of equilibrium. As was shown by D'Aspremont, Gabszewicz, and Thisse (1979), Hotelling's (1929) own model of a spatially differentiated duopoly may fail to possess an equilibrium under certain parameter constellations.

In accordance with Hotelling's (1929) idea, consumer preferences in our model generate each firm's demand as a continuous function of the difference between its own and its rival's price. Through additional assumptions on the distribution of consumer characteristics we ensure that each firm's profit is a quasiconcave function of its price, which guarantees existence of equilibria in pure pricing strategies. Indeed, in price setting games pure strategies are more appealing than mixed strategies if one takes the view that pricing decisions are not irreversible. The reason is that the mixed strategy equilibrium creates some incentives for ex post deviation. In a mixed strategy equilibrium at least some seller can gain by changing his price after learning the realization of the other sellers' prices.²

Interestingly, our framework contains Hotelling's (1929) model as a special case. This allows us to obtain a straightforward insight into the problem of nonexistence of pure price equilibrium. In fact, similar problems arise in a variety of price competition models. In the setting of the present paper the causes of such problems are easily understood and, at least in some cases, it becomes clear which assumptions are required to overcome them.

Following Sattinger (1984) and Perloff and Salop (1985), we assume that there is a continuum of consumers each of whom buys only one of the two brands. Tastes vary within the population and so this approach differs from the representative consumer models of Dixit and Stiglitz (1977) and Spence (1976).³ In contrast with Sattinger (1984) and Perloff and Salop (1985), however, we do not require that the consumers' valuations for the two brands are drawn independently from some probability distribution. In our model, the pattern of tastes within the population may vary in a systematic way and demand need not be symmetric.⁴ As a result, our description of preferences is applicable to a large class of horizontally differentiated

² Dasgupta and Maskin (1986) show that their existence theorems apply to price competition in Hotelling's (1929) model. A more detailed analysis of the mixed strategy equilibrium of this model can be found in Osborne and Pitchik (1988). For the existence of equilibria with discriminatory pricing strategies, see Lederer and Hurter (1986). Novshek (1980) studies an alternative equilibrium concept that drops the assumption of Nash behavior. A bargaining approach to spatial competition is developed in Bester (1989).

³ On the relation between the probability model of consumer preferences and the representative consumer model, see Sattinger (1984), Anderson, De Palma, and Thisse (1987), and Deneckere and Rothschild (1986).

⁴ The symmetry of demand in Sattinger (1984) and Perloff and Salop (1985) allows these authors to focus on single-price equilibria; compare our Proposition 4. Sattinger (1984) and Perloff and Salop (1985) check the second order conditions for profit maximization at the prospective equilibrium point, but do not prove existence of a symmetric equilibrium.

markets including models of spatial competition, in which demand typically fails to be symmetric.

The importance of the distribution of consumer characteristics in models of product differentiation has been recognized in a number of papers that are related to our approach. While we focus on horizontal differentiation, Gabszewicz, Shaked, Sutton, and Thisse (1981) establish conditions on the distribution of consumer incomes that guarantee existence of equilibrium in a model of vertical differentiation. In their model the income distribution determines the shape of the firms' profit functions because richer consumers are willing to pay more for a given improvement in quality. De Palma, Ginsburgh, Papageorgiou, and Thisse (1985) investigate price competition in the logit model of horizontal product differentiation; the basic idea of their approach is to find the appropriate parameter restrictions for a given family of parameterized distribution functions. Champsaur and Rochet (1988) investigate the equilibrium in a model of one-dimensional consumer and product characteristics. Caplin and Nalebuff (1989) do not impose dimensionality restrictions but assume that utility functions are linear with respect to consumer characteristics; in spatial competition models this restricts the applicability of their results essentially to the case of quadratic transportation cost functions. Moreover, their assumptions on the distribution of consumer characteristics are not very appealing in the spatial context because they require the population density over space to be unimodal. The advantage of our approach is that it neither requires a particular functional form of utility or distribution functions nor dimensionality restrictions on product and consumer characteristics. Like most of the literature, we confine ourselves to the case where consumers purchase a single unit of one of the differentiated commodities. Some advances toward divisible commodities are made in Caplin and Nalebuff (1989) and Dierker (1988).

Section 2 of the paper describes the basic model. Section 3 contains the main existence theorem and a discussion of the relationship between product substitutability and price competition. Section 4 provides conditions for the uniqueness of equilibrium. The efficiency properties of the market outcome are studied in Section 5. In Section 6 we compute the equilibrium of an example. Finally, Section 7 shows this example is homeomorphic to Hotelling's (1929) model with quadratic transportation costs. Also, this section provides a simple insight why some models in the literature fail to possess a price setting equilibrium in pure strategies.

2. THE MODEL

We consider a market with two firms, indexed $i = 1, 2$, and a continuum of consumers. Each firm i produces a distinct brand of some commodity at a constant marginal cost c_i .⁵ The duopolists compete by setting prices and p_i is the price charged by firm i . We assume that each consumer needs and buys one unit of either

⁵ With constant marginal costs c_i , firm i has no incentive to ration consumers as long as $p_i \geq c_i$. This allows us to confine ourselves to prices as the firms' strategic variables. In the case of increasing marginal costs, it might be optimal for a firm to restrict its supply and so strategic interactions would become more complex.

the first or the second brand. Effectively this means that the consumers' reservation valuations for the two goods and their incomes are taken to be high enough so that the option of not purchasing a good at all is not relevant within the range of possible equilibrium prices. Indeed, we will show that competition results in equilibrium prices that lie below some upper bound z defined in equation (6) below. Our simplification rules out that the distribution of incomes plays a role in the determination of demand. In our model competition influences each firm's market share but does not affect the total number of sales in the market.

Each consumer is characterized by a preference parameter $\theta \in \mathbb{R}$ that is distributed across the population according to the cumulative distribution function $F(\cdot)$. As we demonstrate below, the parameter θ may be regarded as being derived from some vector of product and preference characteristics. We use the parameter θ as a measure of the intensity by which the consumer prefers the brand supplied by firm 2 to the brand of firm 1. More specifically, consumer θ is willing to spend at most θ more units of income in order to purchase brand 2 rather than brand 1. If $\theta < 0$, the consumer actually prefers the first brand and so he will buy good 2 only if it is cheaper than good 1 by an amount of at least $|\theta|$. Clearly, a consumer with characteristic $\theta = 0$ regards the two brands as perfect substitutes and so he goes to the firm with the lowest price. In summary, consumer θ will buy the good from firm 1 only if

$$(1) \quad \theta \leq p_2 - p_1.$$

We now show that our approach applies to models of purely spatial competition where the two brands are represented by the firms' locations in some geographical space: Let the market area be described by some metric space $\{M, d\}$, where $d(x, y)$ denotes the distance between locations x and y . The characteristics vector of product i is then simply given by the location $x_i \in M$ of firm i . Likewise, each consumer's characteristics are described by his initial location $a \in M$. To purchase good i consumer a has to pay a transportation cost $t(d(x_i, a))$, where $t(\cdot)$ denotes transportation costs as a function of distance d . Consumer a visits firm 1 only if $p_1 + t(d(x_1, a)) \leq p_2 + t(d(x_2, a))$. This is consistent with (1) if we define

$$(2) \quad \theta_a \equiv t(d(x_1, a)) - t(d(x_2, a)).$$

Using (2) we can derive the distribution function $F(\cdot)$ from the distribution of the consumers' initial locations in M . In the location model, therefore, the function $F(\cdot)$ summarizes the consumers' distribution over space, their transportation cost as a function of distances, and the firms' locations.⁶ As an example, in Section 6 we will compute $F(\cdot)$ for Hotelling's (1929) model and demonstrate that our existence result applies if the transportation cost function $t(\cdot)$ lies in some neighborhood of the quadratic function $t(d) = d^2$.

More generally, following Lancaster (1966) and Mas-Colell (1975), each of the two commodities may be represented by a point x_i in some space of commodity characteristics X_i . A good is then described as a bundle of characteristics such as

⁶ The significance of the customer distribution for the existence of equilibrium in Hotelling's (1929) location model is discussed in Shilony (1981).

quality, location, colour, time, and so on. Consumer preferences are defined over the product characteristics vector x and a numeraire commodity m called "money." Preferences vary across consumers and depend on some vector of consumer characteristics $a \in A$. The utility of consumer a is represented by the utility function $U(a, x, m)$. Accordingly consumer a buys good 1 if $U(a, x_1, m - p_1) \geq U(a, x_2, m - p_2)$. To ensure that his decision can be represented by (1) we assume that utilities are quasi-linear in money, i.e. that $U(a, x_1, m) = U(a, x_2, m')$ implies $U(a, x_1, m + \Delta) = U(a, x_2, m' + \Delta)$ for all $\Delta \in \mathbb{R}$. It then follows that consumer a buys good 1 only if $\theta_a \leq p_2 - p_1$ where θ_a is defined as the solution of

$$(3) \quad U(a, x_1, m + \theta_a) = U(a, x_2, m).$$

Note that $F(\cdot)$ depends both upon the product characteristics (x_1, x_2) and the distribution of consumer characteristics a . This contrasts with the approach of Caplin and Nalebuff (1989) where distributional assumptions are made only with regard to a . While we presume knowledge about (x_1, x_2) , Caplin and Nalebuff do not utilize such information.

The consumers' decision rule (1) together with the distribution of preferences determines the market shares of firm 1 and 2 as $F(p_2 - p_1)$ and $1 - F(p_2 - p_1)$, respectively. For each pair of pricing strategies the profits of firm 1 and firm 2 are given as

$$(4) \quad \begin{aligned} \Pi_1(p_1, p_2) &= [p_1 - c_1]F(p_2 - p_1), \\ \Pi_2(p_1, p_2) &= [p_2 - c_2][1 - F(p_2 - p_1)]. \end{aligned}$$

In what follows, we will assume that the distribution of consumer preferences satisfies the following condition.

ASSUMPTION 1. *There is a $\underline{\theta} < 0$ and a $\bar{\theta} > 0$ such that $F(\underline{\theta}) = 0$ and $F(\bar{\theta}) = 1$. Moreover, $F(\cdot)$ is continuous and twice continuously differentiable on $(\underline{\theta}, \bar{\theta})$ with $F'(\theta) > 0$ for all $\underline{\theta} < \theta < \bar{\theta}$.*

Thus the support of $F(\cdot)$ is the compact interval $[\underline{\theta}, \bar{\theta}]$. The firms' profits are continuous functions of their pricing strategies because Assumption 1 precludes atoms in the distribution of θ . As $\underline{\theta} < 0 < \bar{\theta}$, tastes vary in the population and so our analysis is concerned with horizontal, rather than vertical (quality), product differentiation. Indeed, Assumption 1 implies $0 < F(0) < 1$ so that the market share of either firm is positive when both firms quote the same price.

For some part of our analysis the shape of the cumulative distribution function $F(\cdot)$ will be of great importance. As a measure of its concavity we will employ the parameter⁷

$$(5) \quad \rho(\theta) \equiv -F''(\theta)/F'(\theta),$$

⁷ In utility theory this parameter is known as the measure of absolute risk aversion, see Pratt (1964).

which is well defined for all $\theta < \bar{\theta} < \bar{\theta}$. The parameter $\rho(\theta)$ describes the (negative) rate of change of the density at point θ . The steeper the density function at θ , the higher is the absolute value of $\rho(\theta)$.

A market $\{F(\cdot), c_1, c_2\}$ is called *symmetric* if $F(0) = 1/2$ and $c_1 = c_2$. In a symmetric market, equal prices for the two brands result in an equal division of the market and equal profits for the duopolists. The condition $F(0) = 1/2$ is satisfied for instance if the consumers' valuations for each of the two goods are independently drawn from the same probability distribution as in Sattinger (1984) and Perloff and Salop (1986).⁸ It is easy to see that it also holds in location models where consumers are located uniformly on a circle as in Salop (1979). But these markets are special cases and in general the symmetry assumption appears rather strong. For instance, it is easily verified that Hotelling's (1929) location model constitutes a symmetric market only when the duopolists locate their stores at the same distance from the endpoints of the market.⁹

3. PRICE COMPETITION

In this section we analyse price competition under the assumption that the duopolists behave as Nash competitors. A price pair $(p_1^*, p_2^*) \geq (c_1, c_2)$ is called an *equilibrium* of the market $\{F(\cdot), c_1, c_2\}$ if $\Pi_1(p_1^*, p_2^*) \geq \Pi_1(p_1, p_2^*)$ and $\Pi_2(p_1^*, p_2^*) \geq \Pi_2(p_1^*, p_2)$ for all p_1 and p_2 . The restriction $(p_1^*, p_2^*) \geq (c_1, c_2)$ removes weakly dominated pricing strategies from the analysis. In this way we eliminate equilibria in which one of the duopolists charges a price $p_i < c_i$ and receives zero profits because the competitor's price offer attracts the demand of all consumers.

The main result of this section deals with the existence of equilibrium. To state the result, we define the parameter

$$(6) \quad z \equiv \max \left[\frac{\bar{\theta} - \underline{\theta}}{F(0)} + c_2, \frac{\bar{\theta} - \underline{\theta}}{1 - F(0)} + c_1 \right].$$

The proof of the following Proposition reveals that it is always optimal for a firm to quote some price below z as long as the other firm charges a price below z . This fact allows us to compactify the firms' strategy sets. In order to apply a standard fixed point argument, it remains to specify conditions on $F(\cdot)$ that ensure convex-valuedness of the firms' reaction correspondences.

PROPOSITION 1. *Let $-2/(z - c_1) \leq \rho(\theta) \leq 2/(z - c_2)$ for all $\theta < \bar{\theta} < \bar{\theta}$. Then the market $\{F(\cdot), c_1, c_2\}$ has an equilibrium (p_1^*, p_2^*) .*

PROOF. By Assumption 1, $\Pi_i(p_1, p_2)$ is continuous in (p_1, p_2) . We will show that $\Pi_i(p_1, p_2)$ is quasi-concave in p_i as long as $c_i \leq p_i \leq z$. Note that $\Pi_1(p_1, p_2) = 0$ if $p_2 - p_1 \leq \underline{\theta}$ and that $\Pi_1(p_1, p_2)$ is strictly increasing in p_1 if $p_2 -$

⁸ The basic argument that shows that the preference structure in these models satisfies our symmetry definition can be found in footnote 17 of Perloff and Salop (1985).

⁹ Compare Section 6 below.

$p_1 \geq \bar{\theta}$. Therefore, $\Pi_1(p_1, p_2)$ is quasi-concave in p_1 if it is concave in p_1 for $\theta < p_2 - p_1 < \bar{\theta}$. Note that $\Pi_1(p_1, p_2)$ is twice continuously differentiable when $\theta < p_2 - p_1 < \bar{\theta}$. Upon differentiation one obtains

$$(7) \quad \partial^2 \Pi_1(p_1, p_2) / \partial p_1^2 = -2F'(p_2 - p_1) + [p_1 - c_1]F''(p_2 - p_1).$$

Therefore, $p_1 \geq c_1$ and $F''(p_2 - p_1)/F'(p_2 - p_1) \leq 2/(z - c_1)$ imply

$$(8) \quad \partial^2 \Pi_1(p_1, p_2) / \partial p_1^2 \leq 2F'(p_2 - p_1)[p_1 - z]/[z - c_1].$$

By (8) $\partial^2 \Pi_1(p_1, p_2) / \partial p_1^2 \leq 0$ for $p_1 \leq z$ and so $\Pi_1(p_1, p_2)$ is a concave function of p_1 as long as $\theta < p_2 - p_1 < \bar{\theta}$. This proves quasi-concavity of $\Pi_1(p_1, p_2)$; an analogous argument establishes that $\Pi_2(p_1, p_2)$ is quasi-concave in p_2 for $c_2 \leq p_2 \leq z$. As $\Pi_1(p_1, p_2)$ and $\Pi_2(p_1, p_2)$ satisfy the conditions of Theorem 2 in Dasgupta and Maskin (1986), there is a (p_1^*, p_2^*) with $c_i \leq p_i^* \leq z$ such that $\Pi_1(p_1^*, p_2^*) \geq \Pi_1(p_1, p_2^*)$ for all $c_1 \leq p_1 \leq z$ and $\Pi_2(p_1^*, p_2^*) \geq \Pi_2(p_1^*, p_2)$ for all $c_2 \leq p_2 \leq z$.

It thus remains to show that there is no $p_i > z$ such that $\Pi_1(p_1, p_2^*) > \Pi_1(p_1^*, p_2^*)$ or $\Pi_2(p_1^*, p_2) > \Pi_2(p_1^*, p_2^*)$. Suppose there is a $p_1 > z$ such that $\Pi_1(p_1, p_2^*) > \Pi_1(p_1^*, p_2^*)$. If $p_2^* \leq z + \theta$, then $\Pi_1(p_1, p_2^*) = 0$ for all $p_1 > z$, which contradicts $\Pi_1(p_1, p_2^*) > \Pi_1(p_1^*, p_2^*)$. If $z + \theta < p_2^*$, then by offering some price $p_1 > z \geq p_2^*$ firm 1 get the payoff $[p_1 - c_1]F(p_2^* - p_1) \leq [p_1 - c_1]F(0)$ because $F(0) \geq F(p_2^* - p_1)$. Also one must have $p_1 \leq p_2^* - \theta$ because $F(p_2^* - p_1) = 0$ for all $p_1 > p_2^* - \theta$. As a result, by offering $p_1 > z$ firm 1 gets a payoff $\Pi_1(p_1, p_2^*) \leq [p_2^* - \theta - c_1]F(0)$. By setting $p'_1 = p_2^* - \bar{\theta}$ firm 1 could get $p_2^* - \bar{\theta} - c_1$ because then $F(p_2^* - p'_1) = F(\bar{\theta}) = 1$. As a result, setting $p_1 > z \geq p_2^*$ certainly implies $\Pi_1(p_1, p_2^*) \leq \Pi_1(p'_1, p_2^*)$ if $p_2^* - \bar{\theta} - c_1 > [p_2^* - \theta - c_1]F(0)$, i.e., if

$$(9) \quad p_2^* > \frac{\bar{\theta} - \theta}{1 - F(0)} + \theta + c_1.$$

But as $p_2^* > z + \theta$ implies (9), one must have $\Pi_1(p_1, p_2^*) \leq \Pi_1(p_2^* - \bar{\theta}, p_2^*) \leq \Pi_1(p_1^*, p_2^*)$ for all $p_1 > z$, a contradiction. This proves $\Pi_1(p_1^*, p_2^*) \geq \Pi_1(p_1, p_2^*)$ for all p_1 . As symmetric argument proves that also $\Pi_2(p_1^*, p_2^*) \geq \Pi_2(p_1^*, p_2)$ for all p_2 . Q.E.D.

Note that $-2/(z - c_1) < 0 < 2/(z - c_2)$. Therefore, the existence result holds in particular when θ is uniformly distributed over $[\theta, \bar{\theta}]$, because then $\rho(\theta) = 0$ everywhere. In general, Proposition 1 requires $F(\cdot)$ to be neither extremely concave nor extremely convex at any point $\theta < \bar{\theta}$. In other words, the distribution of the preference characteristic must be sufficiently dispersed as to preclude points at which the density function becomes rather steep. As a consequence of this requirement, the impact of small price changes upon the firms' market shares cannot become very large. In the special case of symmetric markets the condition for existence takes the rather simple form $-1/(\bar{\theta} - \theta) \leq \rho(\theta) \leq 1/(\bar{\theta} - \theta)$. Thus the larger the support of $F(\cdot)$, the more demanding becomes the restriction on $\rho(\theta)$. The

inequalities are almost automatically satisfied when all consumers regard the two brands as close substitutes and $\bar{\theta} - \theta$ is close to zero.

The duopolists enjoy a quasi-monopolistic position as long as the customers of each firm regard the brand of the other firm as a poor substitute. The intensity of competition between the two firms should be positively related to the substitutability of their products. In order to study the relationship between product differentiation and competition, we now examine a specific change in the distribution of preferences. Following Perloff and Salop (1985) we multiply each consumer's characteristic θ by some factor $\alpha > 0$. Thus $\alpha > 1$ results in more intense preferences whereas a factor $\alpha < 1$ makes the two brands closer substitutes. In the context of spatial competition models this change of preferences occurs if the original transportation cost $t(\cdot)$ is multiplied by α . Scaling up or down preferences in this way amounts to replacing the distribution function $F(\cdot)$ by the distribution function $F_\alpha(\cdot)$, where

$$(10) \quad F_\alpha(\theta) \equiv F(\theta/\alpha).$$

The following Proposition generalizes a result that was derived by Perloff and Salop (1985) for the case of symmetric markets.

PROPOSITION 2. *Let (p_1^*, p_2^*) be an equilibrium of the market $\{F(\cdot), c_1, c_2\}$ and let $c_1 = c_2$. Then the market $\{F_\alpha(\cdot), c_1, c_2\}$ has an equilibrium (\hat{p}_1, \hat{p}_2) such that $\hat{p}_1 = \alpha p_1^* + (1 - \alpha)c_1$ and $\hat{p}_2 = \alpha p_2^* + (1 - \alpha)c_2$.*

PROOF. By definition of equilibrium we have

$$(11) \quad [p_1^* - c_1]F(p_2^* - p_1^*) \geq [p_1' - c_1]F(p_2^* - p_1'),$$

for all $p_1' \geq c_1$. Using (11), $c_1 = c_2$, and the expressions for (\hat{p}_1, \hat{p}_2) we obtain

$$(12) \quad [\hat{p}_1 - c_1]F_\alpha(\hat{p}_2 - \hat{p}_1) = \alpha[p_1^* - c_1]F(p_2^* - p_1^*) \\ \geq \alpha[p_1' - c_1]F(p_2^* - p_1') = \alpha[p_1' - c_1]F_\alpha(\hat{p}_2 - c_2 + \alpha c_2 - \alpha p_1').$$

If (12) holds for all $p_1' \geq c_1$, then it also holds for all p_1^* defined by $p_1 = \alpha p_1' + (1 - \alpha)c_1$ where $p_1 \geq c_1$. As $c_1 = c_2$, substituting p_1 for p_1' yields

$$(13) \quad [\hat{p}_1 - c_1]F_\alpha(\hat{p}_2 - \hat{p}_1) \geq [p_1 - c_1]F_\alpha(\hat{p}_2 - p_1),$$

for all $p_1 \geq c_1$. Thus \hat{p}_1 satisfies the profit-maximizing condition for firm 1. An analogous argument for firm 2 completes the proof. Q.E.D.

An increase in the factor α raises \hat{p}_i . As $\hat{p}_2 - \hat{p}_1 = \alpha(p_2^* - p_1^*)$ implies $F_\alpha(\hat{p}_2 - \hat{p}_1) = F(p_2^* - p_1^*)$, the firms' market shares remain unaffected. Consequently, in the equilibrium (\hat{p}_1, \hat{p}_2) of the market $\{F_\alpha(\cdot), c_1, c_2\}$ the producers' profits are $\alpha\pi_1(p_1^*, p_2^*)$ and $\alpha\pi_2(p_1^*, p_2^*)$, respectively. An increase in preference intensity raises the profits of both firms. Of course, when the products become perfect substitutes equilibrium profits approach zero in the limit as $\alpha \rightarrow 0$.

4. UNIQUENESS OF EQUILIBRIUM

Having established conditions for the existence of equilibrium, we now turn to the question of whether the equilibrium is unique.

PROPOSITION 3. *Let $\theta < c_2 - c_1 < \bar{\theta}$ and $3/[\theta - (c_2 - c_1)] \leq \rho(\theta) \leq 3/[\bar{\theta} - (c_2 - c_1)]$ for all $\theta < \theta < \bar{\theta}$. Then if there exists an equilibrium (p_1^*, p_2^*) , it is unique.*

PROOF: First it will be shown that $\theta < p_2^* - p_1^* < \bar{\theta}$ in any equilibrium (p_1^*, p_2^*) . Clearly, $\theta \leq p_2^* - p_1^* \leq \bar{\theta}$ in any equilibrium. Thus it remains to show that $\theta \neq p_2^* - p_1^* \neq \bar{\theta}$. Suppose the contrary, for example $p_2^* - p_1^* = \theta$. Then $F(p_2^* - p_1^*) = F(\theta) = 0$ and $\Pi_1(p_1^*, p_2^*) = 0$. But $\Pi_1(p_1^*, p_2^*) = 0$ implies $p_1^* = c_1$. Indeed if $p_1^* > c_1$, then $F(p_2^* - p_1^*) > 0$ for any $c_1 < p_1^* < p_1^*$ so that $\Pi_1(p_1, p_2^*) > 0 = \Pi_1(p_1^*, p_2^*)$, which is inconsistent with the definition of equilibrium. Thus $p_1^* = c_1$ and $p_2^* = p_1^* + \theta$. But then $p_2^* \geq c_2$ implies $c_2 - c_1 \leq \theta$, a contradiction to the conditions of the Proposition. This proves $p_2^* - p_1^* > \theta$. An analogous argument shows $p_2^* - p_1^* < \bar{\theta}$.

As $\theta < p_2^* - p_1^* < \bar{\theta}$, $0 < F(p_2^* - p_1^*) < 1$. Therefore for a sufficiently small $\varepsilon > 0$, also $F(p_2^* - p_1^* - \varepsilon) > 0$. Because $p_1^* + \varepsilon > p_1^* \geq c_1$ implies $0 < \Pi_1(p_1^* + \varepsilon, p_2^*) \leq \Pi_1(p_1^*, p_2^*)$, it must be the case that $p_1^* > c_1$. Similarly, $p_2^* > c_2$. This together with $0 < F(p_2^* - p_1^*) < 1$ implies that (p_1^*, p_2^*) must satisfy the first order conditions for profit-maximization:

$$(14) \quad \partial \Pi_1(p_1, p_2) / \partial p_1 = F(p_2 - p_1) - [p_1 - c_1]F'(p_2 - p_1) = 0$$

$$\partial \Pi_2(p_1, p_2) / \partial p_2 = [1 - F(p_2 - p_1)] - [p_2 - c_2]F'(p_2 - p_1) = 0.$$

Define $\theta^* \equiv p_2^* - p_1^*$ and

$$(15) \quad \psi(\theta) \equiv 2F(\theta) + [\theta - (c_2 - c_1)]F'(\theta) - 1.$$

Then by subtracting the two equations in (14) we find that if (p_1^*, p_2^*) is a solution to the first-order conditions, θ^* must satisfy $\psi(\theta^*) = 0$. We will show that θ^* is unique because $\psi(\cdot)$ is strictly increasing over $(\theta, \bar{\theta})$.

Notice that

$$(16) \quad \psi'(\theta) = 3F'(\theta) + F''(\theta)[\theta - (c_2 - c_1)].$$

Consider the set $H_1 \equiv \{\theta | \theta < \theta < c_2 - c_1\}$. Then by the conditions of the Proposition, $3/[\theta - (c_2 - c_1)] < 3/[\theta - (c_2 - c_1)] \leq -F''(\theta)/F'(\theta)$ for all $\theta \in H_1$. Thus $F''(\theta)[\theta - (c_2 - c_1)] > -3F'(\theta)$ for all $\theta \in H_1$ which yields $\psi'(\theta) > 0$ for all $\theta \in H_1$.

Now consider the set $H_2 \equiv \{\theta | c_2 - c_1 < \theta < \bar{\theta}\}$. Then $3/[\theta - (c_2 - c_1)] > 3/[\bar{\theta} - (c_2 - c_1)] \geq -F''(\theta)/F'(\theta)$. Accordingly $F''(\theta)[\theta - (c_2 - c_1)] > -3F'(\theta)$ for all $\theta \in H_2$, implying $\psi'(\theta) > 0$ for all $\theta \in H_2$.

As $\psi'(\theta) > 0$ for all $\theta \in H_1 \cup H_2$, $\psi(\cdot)$ is strictly increasing over $(\theta, \bar{\theta})$. As a result $\theta^* = p_2^* - p_1^*$ is unique. It then follows from (14) that also (p_1^*, p_2^*) is uniquely determined. Q.E.D.

To ensure uniqueness of the equilibrium, the producers' cost differences have to be small relative to the size of the support of $F(\cdot)$. In addition, consumer preferences have to be sufficiently dispersed. Again, the uniform distribution is a particular example satisfying the conditions of $\rho(\theta)$. Proposition 3 is important for extending the present analysis to a two-stage game in which the duopolists first simultaneously decide on the characteristics x_i of their product and then compete by setting prices. Given uniqueness of the second-stage outcome, the firms' first-stage payoffs are well-defined functions of their choices x_1 and x_2 . Indeed, using Propositions 1 and 3 we can briefly outline the arguments to prove existence of a subgame perfect equilibrium in the two-stage game. To indicate the dependence of the preference distribution on (x_1, x_2) , let $F(\theta, x_1, x_2)$ denote the distribution function for a given choice of x_1 and x_2 . Similarly, firm i 's unit cost now becomes a function $c_i(x_i)$ of product characteristics. Assume that each firm i chooses x_i from some convex and compact set $X_i \subset \mathbb{R}^m$ with $X_1 \cap X_2 = \emptyset$.¹⁰ Moreover let $F(\theta, \cdot)$ and $c_i(\cdot)$ be continuous in (x_1, x_2) . If then the conditions of Propositions 1 and 3 are satisfied, a simple continuity argument establishes that the equilibrium (p_1^*, p_2^*) and the equilibrium payoffs $\Pi_1(p_1^*, p_2^*)$ and $\Pi_2(p_1^*, p_2^*)$ in the second-stage subgame depend continuously on (x_1, x_2) . Accordingly, we can apply Theorem 3 of Dasgupta and Maskin (1986) which proves that the first-stage game possesses a mixed strategy equilibrium in which each firm i randomizes over X_i . Thus the overall game has a pure strategy equilibrium in the second stage and a mixed strategy equilibrium in the first stage.¹¹ This makes sense if one takes the view that, in contrast with pricing decisions, production decisions exhibit some degree of irreversibility so that mixed strategies in the first game stage are sensible.

In general one cannot conclude whether the conditions of Proposition 1 are more restrictive than those of Proposition 3 or vice versa. Indeed, the parameter z in Proposition 1 depends upon $F(0)$ whereas the value of $F(0)$ plays no role in Proposition 3. For the case of symmetric markets, however, it is easily verified that the existence result also implies uniqueness. Thus, as a Corollary to Propositions 1 and 3 we obtain¹²:

PROPOSITION 4. *Let $-1/[\bar{\theta} - \theta] \leq \rho(\theta) \leq 1/[\bar{\theta} - \theta]$ for all $\theta < \theta < \bar{\theta}$. Then if $\{F(\cdot), c_1, c_2\}$ is a symmetric market, it has a unique equilibrium (p_1^*, p_2^*) and $p_1^* = p_2^*$.*

PROOF. By symmetry $F(0) = 1/2$ and $c_1 = c_2$. Therefore $2/(z - c_1) = 2/(z - c_2) = 1/[\bar{\theta} - \theta]$. Thus the conditions of Proposition 1 are satisfied and there is an equilibrium. As $3/\theta < -1/[\bar{\theta} - \theta]$ and $1/[\bar{\theta} - \theta] < 3/\bar{\theta}$, also the conditions of Proposition 3 are satisfied. Thus there is a unique equilibrium (p_1^*, p_2^*) . In

¹⁰ The condition $X_1 \cap X_2 = \emptyset$ ensures that Assumption 1 is applicable. If $x_1 = x_2$, then the two products are no longer differentiated and so $F(\cdot, x_1, x_2)$ is degenerate. To allow for $X_1 \cap X_2 \neq \emptyset$ one can simply set $\Pi_1(p_1^*, p_2^*) = \Pi_2(p_1^*, p_2^*) = 0$ for $x_1 = x_2$ and use a continuity argument showing that both firms' payoffs tend to zero when their products become identical.

¹¹ Interestingly, this contrasts with Osborne and Pitchik (1987) who analyze Hotelling's (1929) model with a mixed equilibrium in the price setting game and a pure equilibrium in the location game.

¹² The uniqueness of the equilibrium in the symmetric duopoly has also been noted by Perloff and Salop (1985).

addition, $\theta^* = p_2^* - p_1^*$ is the unique solution of $\psi(\theta) = 0$, where $\psi(\cdot)$ is defined as in (15). Using $F(0) = 1/2$ and $c_1 = c_2$ it is easily checked that $\psi(0) = 0$. This proves $\theta^* = p_2^* - p_1^* = 0$. Q.E.D.

5. EFFICIENCY

In this section we will analyze the equilibrium from the viewpoint of social efficiency. In the social optimum both brands should be produced only if $\theta < c_2 - c_1 < \bar{\theta}$. To see this, consider the case $c_2 > c_1$. Brand 2 should then be supplied only if there are consumers who are willing to bear the extra cost $c_2 - c_1$ for substituting good 1 by good 2. This is the case if the set $\{\theta | \theta \geq c_2 - c_1\}$ has positive measure or, equivalently, if $\bar{\theta} > c_2 - c_1$. By an analogous argument, brand 1 should be made available only if $\theta < c_2 - c_1$. The following proposition shows that the market outcome may involve positive profits for both firms even under parameter constellations where producing both brands is socially inefficient.

PROPOSITION 5. *Let $\theta/F(0) < c_2 - c_1 < \bar{\theta}/[1 - F(0)]$. Then $\Pi_1(p_1^*, p_2^*) > 0$ and $\Pi_2(p_1^*, p_2^*) > 0$ in any equilibrium (p_1^*, p_2^*) .*

PROOF. First it will be shown that $p_1^* > c_1$. Suppose the contrary, i.e. $p_1^* = c_1$. This implies $p_2^* = c_1 + \theta$. Indeed, one cannot have $p_2^* < c_1 + \theta$ because $F(p_2 - c_1) = 0$ for all $p_2 < c_1 + \theta$ so that $\Pi_2(c_1, c_1 + \theta) > \Pi_2(c_1, p_2)$. Similarly, one cannot have $p_2^* > c_1 + \theta$ because then $F(p_2^* - p_1) > 0$ and $\Pi_1(p_1, p_2^*) > \Pi_1(p_1^*, p_2^*) = 0$ for all $c_1 < p_1 < p_2^* - \theta$. Thus $p_1^* = c_1$ implies $p_2^* = c_1 + \theta$. Accordingly

$$(17) \quad \Pi_2(p_1^*, p_2^*) = c_1 + \theta - c_2.$$

By setting $p_2 = c_1$ firm 2 could get $\Pi_2(p_1^*, c_1) = \Pi_2(c_1, c_1) = [c_1 - c_2][1 - F(0)]$. But then $[c_2 - c_1]F(0) > \theta$ implies $\Pi_2(p_1^*, c_1) > \Pi_2(p_1^*, p_2^*)$, a contradiction.

Next it will be shown that $p_1^* > c_1$ implies $\Pi_1(p_1^*, p_2^*) > 0$. Suppose the contrary, i.e., $\Pi_1(p_1^*, p_2^*) = 0$. Then $p_1^* > c_1$ implies $F(p_2^* - p_1^*) = 0$ so that $p_2^* - p_1^* \leq \theta$. As $\Pi_2(p_1^*, p_1^* + \theta) > \Pi_2(p_1^*, p_2)$ for all $p_2 < p_1^* + \theta$ this implies $p_2^* = p_1^* + \theta$. But then one has $0 = \Pi_1(p_1^*, p_2^*) = \Pi_1(p_1^*, p_1^* + \theta) < \Pi_1(p_1, p_1^* + \theta)$ for all p_1 such that $c_1 < p_1 < p_1^* = p_2^* - \theta$, a contradiction. This proves $\Pi_1(p_1^*, p_2^*) > 0$. An analogous argument shows that $[c_2 - c_1][1 - F(0)] < \bar{\theta}$ implies $\Pi_2(p_1^*, p_2^*) > 0$. Q.E.D.

Note that the interval $(\theta/F(0), \bar{\theta}/[1 - F(0)])$ contains the interval $(\theta, \bar{\theta})$ as a subset because $0 < F(0) < 1$. When the brand specification of the two firms is fixed, the equilibrium may involve too much but not too little product variety.

Using Proposition 5 we can easily construct an environment in which excessive product differentiation also occurs when the firms are free to choose their products' characteristics. Consider a two-stage game in which the firms first decide on the variant of their product and then compete by setting prices. For simplicity, assume that firm 2 can supply only one type of brand at the unit cost c_2 . Firm 1, however, has two options: It can either produce some commodity a at the cost c_1^a or another

commodity b at the cost c_1^b . If firm 1 selects good a , then $F(\cdot)$ denotes the cumulative distribution of the preference characteristic $\theta \in [\underline{\theta}, \bar{\theta}]$; if it selects good b , the corresponding distribution function is $F_a(\cdot)$, as defined by (10). Now assume $\theta/F(0) < c_2 - c_1^a < \bar{\theta}$ and $c_2^b = c_1$. Clearly, the efficiency criterion requires firm 1 to produce good b rather than good a . Yet, it follows from Propositions 2 and 4 that choosing brand a yields higher profits for firm 1 in the price setting subgame whenever α is close to zero. In this example, firm 1 will use product differentiation to relax price competition, a phenomenon that has been observed in spatial competition by D'Aspremont, Gabszewicz, and Thisse (1979) and in quality competition by Shaked and Sutton (1982).

Even when operating both firms is socially optimal, the equilibrium allocation of the two goods may turn out to be inefficient. According to the above efficiency argument, all consumers with characteristic $\theta < c_2 - c_1$ should consume the first brand while those with taste parameter $\theta > c_2 - c_1$ should buy the second brand. In the market equilibrium, however, the buyers' decisions are determined by the price differential $p_2^* - p_1^*$. Consumer θ purchases good 1 only if $\theta \leq p_2^* - p_1^*$; otherwise he buys good 2. Consequently, the market allocation of the two brands is efficient only if $p_2^* - p_1^* = c_2 - c_1$. That is, the profit margins of both products have to be identical. As the following Proposition shows, however, oligopolistic competition will typically fail to satisfy this condition.

PROPOSITION 6. *Let (p_1^*, p_2^*) be an equilibrium such that $\Pi_1(p_1^*, p_2^*) > 0$ and $\Pi_2(p_1^*, p_2^*) > 0$. Then $p_1^* - c_1 = p_2^* - c_2$ only if $F(c_2 - c_1) = 1/2$. If $F(c_2 - c_1) > 1/2$ then $p_1^* - c_1 > p_2^* - c_2$, and if $F(c_2 - c_1) < 1/2$ then $p_1^* - c_1 < p_2^* - c_2$.*

PROOF. As $\Pi_1(p_1^*, p_2^*) > 0$ and $\Pi_2(p_1^*, p_2^*) > 0$, (p_1^*, p_2^*) must satisfy the first order conditions (14) and $\theta^* = p_2^* - p_1^*$ must solve $\psi(\theta) = 0$, where $\psi(\cdot)$ is defined as in (15). Thus $\theta^* = c_2 - c_1$ and $F'(\theta^*) > 0$ immediately implies $2F(\theta^*) = 2F(c_2 - c_1) = 1$.

To prove the second statement suppose the contrary, i.e. $F(c_2 - c_1) > 1/2$ and $\theta^* \geq c_2 - c_1$. This implies $2F(\theta^*) > 1$. But $2F(\theta^*) > 1$ and $\theta^* \geq c_2 - c_1$ is inconsistent with $\psi(\theta^*) = 0$, a contradiction. This proves that $F(c_2 - c_1) > 1/2$ implies $p_2^* - p_1^* < c_2 - c_1$. An analogous argument shows that $F(c_2 - c_1) < 1/2$ implies $p_2^* - p_1^* > c_2 - c_1$. Q.E.D.

Consequently, too many consumers buy good 2 and firm 1's market share is inefficiently low whenever $F(c_2 - c_1) > 1/2$. If $F(c_2 - c_1) < 1/2$, the inefficiency is reversed. Proposition 6 also indicates that symmetric markets have a particular efficiency property: The profit margins of both commodities are identical and create no distortions in the consumers' purchasing decisions.

6. AN EXAMPLE

In this section we compute the equilibrium (p_1^*, p_2^*) of a market with a uniform distribution of the taste parameter θ . Moreover, as we will show in the following

section, this example corresponds to Hotelling's (1929) spatial duopoly in the case of quadratic transportation costs.

Let $F(\theta) = [\theta - \bar{\theta}]/[\bar{\theta} - \theta]$ for $\theta \leq \theta \leq \bar{\theta}$. Then the first order condition for profit maximization shows that firm 1 reacts optimally to firm 2's price p_2 by setting $p_1 = c_1$ if $p_2 \leq c_1 + \theta$; $p_1 = 0.5[p_2 - \theta + c_1]$ if $c_1 + \theta \leq p_2 \leq c_1 - \theta + 2\bar{\theta}$; and $p_1 = p_2 - \bar{\theta}$ if $p_2 \geq c_1 - \theta + 2\bar{\theta}$. Similarly, profit maximization by firm 2 implies setting $p_2 = c_2$ if $p_1 \leq c_2 - \bar{\theta}$; $p_2 = 0.5[p_1 + \bar{\theta} + c_2]$ if $c_2 - \bar{\theta} \leq p_1 \leq c_2 + \bar{\theta} - 2\theta$; and $p_2 = p_1 + \theta$ if $p_1 \geq c_2 + \bar{\theta} - 2\theta$.

Given these reaction functions, it follows that the equilibrium is unique. Depending upon the cost differential $c_2 - c_1$, there are three possible categories of equilibrium: If $c_2 - c_1 \geq 2\bar{\theta} - \theta$, then only firm 1 is active in equilibrium. The equilibrium prices are $p_1^* = c_2 - \bar{\theta}$, $p_2^* = c_2$ so that $\Pi_1(p_1^*, p_2^*) = c_2 - \bar{\theta} - c_1$ and $\Pi_2(p_1^*, p_2^*) = 0$. If $2\theta - \bar{\theta} < c_2 - c_1 < 2\bar{\theta} - \theta$, then both firms are active. The equilibrium prices are given as

$$(18) \quad p_1^* = \frac{2}{3}[c_1 - \theta] + \frac{1}{3}[c_2 + \bar{\theta}], \quad p_2^* = \frac{2}{3}[c_2 + \bar{\theta}] + \frac{1}{3}[c_1 - \theta];$$

and the corresponding equilibrium payoffs are

$$(19) \quad \Pi_1(p_1^*, p_2^*) = \frac{[c_2 - c_1 + \bar{\theta} - 2\theta]^2}{9[\bar{\theta} - \theta]},$$

$$\Pi_2(p_1^*, p_2^*) = \frac{[c_1 - c_2 - \theta + 2\bar{\theta}]^2}{9[\bar{\theta} - \theta]}.$$

Finally, only firm 2 is active if $c_2 - c_1 \leq 2\theta - \bar{\theta}$. In this case $p_1^* = c_1$ and $p_2^* = c_1 + \theta$. Accordingly, $\Pi_1(p_1^*, p_2^*) = 0$ and $\Pi_2(p_1^*, p_2^*) = c_1 + \theta - c_2$.

7. APPLICATIONS

In this section we will review Hotelling's (1929) model of duopolistic spatial competition in the light of our findings. Our approach provides an easy understanding of why this model fails to have a pure strategy equilibrium in the case of linear transportation cost. Also, it demonstrates how changes in the transportation cost function may restore existence of such an equilibrium. In addition, we show that our framework can be used to explain nonexistence of pure strategy equilibrium in Shilony's (1977) model of mixed pricing in oligopoly or Varian's (1980) model of sales.

Hotelling's (1929) model can easily be translated into our description of a market by a tuple $\{F(\cdot), c_1, c_2\}$. In his model the population of consumers is uniformly distributed over the interval $[0, 1]$. The two firms offer products that are identical in all respects except for the location of availability. The latter is determined by the locations $x_1 \in [0, 1]$ and $x_2 \in [0, 1]$ of firm 1 and 2, respectively, where $x_1 < x_2$. The consumer faces a transportation cost that is a function $t(\cdot)$ of (Euclidian) distance. For the consumer who is located at $a \in [0, 1]$ the overall cost of purchasing brand i is then $p_i + t(|x_i - a|)$. Consequently, this consumer will visit

firm 1 only if $t(|x_1 - a|) - t(|x_2 - a|) \leq p_2 - p_1$. Therefore, it follows from (1) that this consumer's preference parameter θ_a is given as

$$(20) \quad \theta_a \equiv t(|x_1 - a|) - t(|x_2 - a|).$$

As a is uniformly distributed over $[0, 1]$, (20) allows us to compute the cumulative distribution function $F(\cdot)$ of the parameter θ .

Hotelling (1929) took $t(\cdot)$ to be linear so that $t(|x_i - a|) = k|x_i - a|$ with $k > 0$. It then follows from (20) that all consumers located in the interval $[0, x_1]$ have the same preference characteristic $\theta = k(x_1 - x_2)$. If a lies in the interval (x_1, x_2) , then the associated taste parameter is $\theta = k(2a - x_1 - x_2)$. Finally, all consumers in the interval $[x_2, 1]$ have the same characteristic $\theta = k(x_2 - x_1)$. In summary, the assumption of linear transportation costs generates a preference distribution with support $[\underline{\theta}, \bar{\theta}] = [k(x_1 - x_2), k(x_2 - x_1)]$ and a cumulative distribution function

$$(21) \quad F(\theta) = \begin{cases} 0, & \text{if } \theta < \underline{\theta}, \\ 0.5(x_1 + x_2 + \theta/k), & \text{if } \underline{\theta} \leq \theta < \bar{\theta}, \\ 1, & \text{if } \theta \geq \bar{\theta}. \end{cases}$$

Clearly, this distribution violates our Assumption 1 because there are mass points at the lower and upper end of the support $[\underline{\theta}, \bar{\theta}]$ whenever $x_1 > 0$ and $x_2 < 1$. These mass points generate discontinuities in demand. Even more importantly, they preclude approximating $F(\cdot)$ by a sequence of continuous distributions that satisfy the conditions of Proposition 1. As a result, with linear transportation costs the duopolists' payoffs may fail to be quasi-concave so that an equilibrium may not exist. In fact, D'Aspremont, Gabszewicz, and Thisse (1979) pointed out that this nonexistence problem becomes relevant when the distance $x_2 - x_1$ between the two stores is relatively small.

D'Aspremont, Gabszewicz, and Thisse (1979) also showed that a quadratic cost function $t(|x_i - a|) = k(x_i - a)^2$ results in the existence of equilibrium for any given locations (x_1, x_2) of the two firms. To see how this change in transportation costs alters the distribution function $F(\cdot)$, note that in the quadratic case $\partial \theta / \partial a = 2k(x_2 - x_1) > 0$ and so $\underline{\theta} = k[x_1^2 - x_2^2]$ and $\bar{\theta} = k[(1 - x_1)^2 - (1 - x_2)^2]$. Moreover, as θ is a linear function of a , it follows from the uniform distribution of a over $[0, 1]$ that θ is uniformly distributed over $[\underline{\theta}, \bar{\theta}]$. Consequently, the prices (p_1^*, p_2^*) and profits $\Pi_1(p_1^*, p_2^*)$ and $\Pi_2(p_1^*, p_2^*)$ of our above example describe the equilibrium of Hotelling's model when transportation costs are quadratic. In particular if $c_1 = c_2$, both firms are active and our equations (18) and (19) coincide with the equilibrium computed by D'Aspremont, Gabszewicz, and Thisse (1979).

Interestingly, our existence result goes beyond the case of quadratic transportation costs: Consider a sequence of cost functions $\{t_n(d)\}_{n=1}^\infty$ such that $\lim_{n \rightarrow \infty} t_n(d) = d^2$ for all $0 \leq d \leq 1$. A simple continuity argument then establishes that Theorem 1 holds for n large enough. That is, existence of equilibrium is guaranteed as long as transportation costs are not too far from being quadratic.

Our approach reveals that mass points in the preference distribution are

responsible for the nonexistence of pure Nash equilibria not only in the original Hotelling model but also in the oligopolistic pricing model of Shilony (1977) or in Varian's (1980) model of sales. In Varian's model there are informed and uninformed consumers. The former buy the good at the store with the lowest price, whereas the latter randomly decide whether to shop at store 1 or store 2. Clearly, with these assumptions the preference distribution must violate our continuity assumption which explains why there is only a mixed equilibrium in Varian's model. Similarly, Shilony considers a market where consumers can purchase costlessly from a neighborhood store, but incur some cost s if they venture a more distant store. In our terminology this means that all consumers in the neighborhood of firm 1 have the characteristic $\theta = \underline{\theta} = -s$ as they stay with firm 1 whenever $-s \leq p_2 - p_1$. Similarly, consumers in the neighborhood of firm 2 are willing to visit firm 1 only if $s \leq p_2 - p_1$ and so they have the characteristic $\theta = \bar{\theta} = s$. As Shilony demonstrates, his model fails to have a pure price equilibrium. In our framework this is easily understood as it generates a distribution function characterized by $F(\theta) = 0.5$ for $\underline{\theta} \leq \theta < \bar{\theta}$ with two point masses of 0.5 at $\underline{\theta}$ and $\bar{\theta}$. Our analysis also indicates that a dispersion of the cost s can restore existence of a pure price setting equilibrium. Assume for example that s is uniformly distributed on $[0, \bar{s}]$ across the population of consumers. Then this is easily seen to imply a uniform distribution of θ on $[\underline{\theta}, \bar{\theta}]$ with $\underline{\theta} = -\bar{s}$ and $\bar{\theta} = \bar{s}$. We may thus conclude that this modification implies existence of a unique Nash equilibrium in pure strategies.

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